Structural adjustment, job turnover and career progression

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Abstract

We develop a dynamic, stochastic, multi-sectoral, equilibrium model that allows for worker turnover, job turnover and career mobility. This serves to bridge the literatures on job reallocation and career progression. Our model makes a number of predictions: a positive correlation between job turnover rates and promotion rates; a positive correlation across sectors between mean real income and their corresponding variance; an inverse relationship between sector profitability and both the job turnover rate and income inequality. These predictions are supported empirically.

Keywords: Worker turnover, job turnover, career mobility

JEL classification: J62, J63, J24
1 Introduction

Our main purpose in this paper is to build a labour market model that captures the intense job reallocation patterns identified by Davis, Haltiwanger & Schuh (1996), and the complex worker reallocation patterns which have been recognised and studied for rather longer; see for example Farber (1999). Crucially, we allow workers to move not only between jobs and sectors, but also between non-managerial and managerial positions. Thus workers have “careers” which are affected not only by their own decisions, but also by the decisions of firms. This contrasts with the extant literature on career mobility, which typically considers only the decisions of one side of the market, as discussed in Section 2.

The model will allow us to assess the impact of structural shocks on worker turnover, job turnover, relative wages and workers’ promotion patterns.

The patterns of job reallocation in advanced economies are well-known: see Davis & Haltiwanger (1999) for a detailed summary. The total job reallocation rate (the sum of job creation and destruction) is typically many times greater than net employment change. Thus, even when the total number of jobs is roughly constant, there are large numbers of jobs being created and destroyed. This is even true within narrowly-defined sectors of the economy, so the observed reallocation of jobs is not the result of sectoral reallocation.

Patterns of worker turnover are rather more complex, and to illustrate these we use a representative panel of workers in the UK from the British Household Panel Survey (BHPS).

The first row of Table 1 shows that only 74% of those individuals in private sector employment were working for the same firm in the next year; the remaining 26% moved to another firm or non-employment.

1Detailed information on the BHPS is available from http://www.iser.essex.ac.uk/ulsc/bhps/; see also Taylor, Brice, Buck & Prentice-Lane (2006). The data is freely available to academic researchers and the code which produced our estimates is available from the authors on request.
Remains
in firm

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>0.74</td>
<td>0.26</td>
</tr>
<tr>
<td>Promoted</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>Same-level</td>
<td>0.87</td>
<td>0.79</td>
</tr>
<tr>
<td>Demoted</td>
<td>0.05</td>
<td>0.13</td>
</tr>
<tr>
<td>Voluntary</td>
<td>—</td>
<td>0.75</td>
</tr>
<tr>
<td>Involuntary</td>
<td>—</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 1: Worker reallocation in the UK
1991–2005

about 8% are promoted from non-managerial to managerial or supervisory roles. A smaller number of those who remain are demoted. Amongst those who move to a new firm, rates of promotion are comparable but demotion rates are considerably higher. The bottom panel shows the proportion of individuals who move voluntarily or involuntarily.

Of those who move firm 25% report that they left their last job involuntarily.

The key features of the labour market which we wish to model are based on the stylised facts we have just discussed:

1. There are large job flows in equilibrium, even within sectors;
2. Worker flows are necessarily larger than job flows because workers may move around a given set of jobs;
3. There is considerable career mobility, both up and down the ladder;
4. Workers move for both voluntary and involuntary reasons.

We introduce both worker and firm heterogeneity in a dynamic, stochastic, multi-sectoral, equilibrium matching model of the labour market. There are two high-tech sectors that

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2The BHPS asks individuals if they have any managerial duties or if they supervise other employees. We use this information to determine whether individuals have been promoted or demoted.

3The BHPS asks individuals who changed jobs between interview dates why they left their previous jobs. We code someone as having left involuntarily if they respond “Made redundant”, “Dismissed/sacked” or “Temporary job ended”.

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employ both managerial and blue-collar workers and a low-tech sector where only operatives are required.

Job turnover occurs because firms are subject to idiosyncratic productivity shocks. New jobs are created as firms enter the market, while other jobs are destroyed because previously established firms exit. Because productivity shocks are firm-specific, we observe job turnover in equilibrium: while the number of active firms in each sector is constant, the identity of active firms changes.

Skills come in different forms and are captured by three worker characteristics: firm match, sectoral match and ability, whose exogenous distributions capture the heterogeneity of firm-specific and general skills in the market.

In our model, worker mobility has many dimensions. At any point in time some workers change jobs without moving to another firm by being promoted. Others move to new firms but they stay in the same sector. Still others find employment in new sectors. Some of these moves are initiated by the workers themselves as they look for employers that can better match their abilities. The rest of the moves are involuntary from the workers’ point of view. Involuntary separations occur if firms look for better-matched workers, or if firms exit the market.

Given the complicated nature of the labour market in our model, we use a market-clearing rather than a search framework to make the analysis tractable. Therefore we do not observe unemployment or search. Recent evidence by Nagypal (2005, 2008) strongly suggests that job-to-job transitions are more important for understanding the dynamics of the labor market and job mobility than transitions from employment to unemployment. Nagypal (2005) further argues that most standard search models that allow for on-the-job search

\[4\] Firms are also assumed to have a fixed employment size and thus we ignore job turnover that occurs within existing firms: in reality, some firms expand and create new jobs while others contract their operations and destroy jobs. This type of job mobility has been the subject of a number of papers that study the evolution over time of the distribution of firm size within an industry (e.g. Jovanovic 1982, Caballero & Hammour 1994, Ericson & Pakes 1995, Hopenhayn 1992).
cannot account for the extent of the job-to-job transitions. Our model provides an alterna-
tive to the search approach but does so at a cost since it does not allow for unemployment.
However, while there is no explicit role for unemployment in our model, an alternative
interpretation of the low-tech sector could be that it represents unemployment, where the
“wage” is the value of leisure or unemployment benefit.

There is also underemployment of skills, because some high-ability workers end up with
non-managerial jobs, and some blue-collar workers are not matched with firms in sectors
which best-suit their skills. The main advantage of our approach is that we can perform
a number of comparative static experiments and compare them in a unified framework.
For example, we are able to distinguish between a skill-biased technological change from
one that affects all workers uniformly, and a sectoral shock from an economy-wide one.
We can also assess how changes in the distributions of worker characteristics affect worker
and job mobility.

Our model makes a number of novel predictions which can be tested empirically. First,
job turnover and promotion rates are positively correlated; this occurs in part because job
turnover creates opportunities for workers to climb the career ladder. Second, our model
predicts a positive correlation between the average wage in a sector and the variance of
wages. Third, we show how changes in the basic parameters of our model can represent
shocks which affect certain sectors, and shocks which affect certain types of worker. We
can show how these shocks affect job turnover, worker turnover, promotion rates, relative
wages and underemployment.

The analysis contained in this paper bridges several existing literatures, which are discussed
in Section 2. Section 3 then sets out the model, explains why the particular structure has
been adopted, and then characterises the equilibrium of the model. Section 4 discusses
the comparative statics of the model and examines, in particular, the impact of structural
change on the career progression of individuals. Section 5 concludes.
2 Previous Literature

The literature that studies the problem of matching jobs that differ in their skill requirements with workers that differ in skills is an old one and is reviewed in Sattinger (1993). In most matching models learning on both sides of the market leads to improved employer/employee partnerships and, if the economy is left undisturbed, an optimal allocation would be achieved in the long-run. Shimer (2007) advances an alternative view according to which “mismatch” is the prevailing characteristic of labour markets. Even if the labour market clears at each instant, because some markets have more workers than jobs and other more jobs than workers, unemployment and vacancies coexist in equilibrium. This is also a property of our model, in which not all high-ability workers can find employment in the two high-tech sectors where their managerial skills are useful and they end up obtaining employment in the operative sector. The quality of a match and thus its likelihood of survival depends on both worker and firm characteristics. On the worker side, the firm and sectoral match and level of ability depend on the relative acquisition of specific and general skills. Thus our model integrates supply-side considerations that have been emphasized in the extensive literature on skill acquisition and its relation to worker mobility, surveyed in Malcomson (1997) and Leuven (2004).

Much of the modern literature on career mobility stems from the work of Sicherman & Galor (1990). They consider how fully-informed, forward-looking, agents choose their optimal human capital investment, and subsequently their optimal career path, in order to maximise their lifetime income. This serves as a useful extension to the basic model of human capital investment. However, although the individual’s career may involve movements up and across occupational ladders, the demand side in this model is taken as given. Hence workers progress smoothly, and with perfect foresight, along their chosen career paths with no prospect of being knocked off or back down their chosen career ladder.

Recent empirical studies on the importance of occupational mobility include Kambourov &
Manovskii (2008a, 2009), who document increasing occupational and industrial mobility in the US, and the fact that this increasing mobility may be a significant factor in explaining increases in wage inequality. Meanwhile, Moscarini & Thomson (2007) demonstrate the importance of within-firm occupational moves — about 40% of occupational moves are internal to an employer.

There is a fast-growing literature on dynamic equilibrium matching models with both worker and firm heterogeneity that is related to our work, including Acemoglu (1999, 2001), Albrecht & Vroman (2002), Burdett & Coles (1999), Burdett & Mortensen (1998), Kiyotaki & Lagos (2007), Mortensen & Pissarides (1999) Sattinger (1995), Shi (2002), Shimer (2005) and Shimer & Smith (2000). The emphasis of these search models is predominantly on the consequences of skill-biased technological change for wages and unemployment. The main advantage of our work is that it also considers the impact of technological changes on intra-firm and cross-sectoral mobility. Our work is also related to the job creation and job destruction literature that employs general equilibrium multi-sectoral models. Most of the work in this area extends the standard search and matching framework and includes Davidson, Martin & Matusz (1987, 1988), Hosios (1990) and Greenwood, MacDonald & Zhang (1996). Not incorporating search in our theory implies that we cannot address sectoral unemployment issues. However, our approach allows us to dispense with the matching function and use a more disaggregated approach that yields rich worker and job mobility patterns. Hopenhayn & Rogerson (1993) and Prescott (1992) use the market-clearing framework and their work belongs to the family of stochastic general equilibrium real business cycles models. Markets also clear in our model, however, the product demand side is exogenous. The advantage of our approach is that it allows for greater heterogeneity on both sides of the labour market.

There is also a voluminous literature which examines the wage and employment conse-

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\[\text{See Mortensen and Pissarides (1999b) and Rogerson, Shimer and Wright (2005) for reviews of the search-theoretic literature.}\]
quences of economy-wide structural change on individuals. Much of this has been framed in terms the relative impact of trade (e.g. Wood 1995) or technology (e.g. Berman, Bound & Machin 1998). Although this debate is by no means closed, the declining job prospects of low-skilled workers has been attributed by many authors to the prevalence from the 1980s of skill-biased technological change. Our work contributes to this literature by considering, in addition, how such shocks might impact on the career paths of individuals as well as their wages. Finally, although the empirical literature has noted the increased prevalence of skilled occupations in the workforce, there is relatively little research which examines how firms (or industries) use their hiring, firing and, in particular, their internal promotion decisions in order to upgrade their workforce. Some indirect evidence suggests that this may be important. Haskel & Heden (1999) find that the exit and entry of new plants contributes between 34% and 45% of the increase in the proportion of skilled employment in the UK, with Dunne, Haltiwanger & Troske (1997) finding for the US that the entry and exit of plants contributes about 36% to the increase in the proportion of non-production employment. This implies that about two thirds of skill upgrading occurs within or between existing plants. It is likely that a significant proportion of within firm upgrading is by the promotion of existing workers, though how this compares in importance to the hiring of outsiders is unknown. The related literature on training provision suggests that internal promotion may be important in the process of skill upgrading. Bartel & Sicherman (1998) show that individuals in industries with higher rates of technological change are more likely to receive training. If, as seems likely, those in receipt of training have a higher probability of movement to higher skill levels, then technological change and career progression will be positively correlated.
3 The Model

We envisage a labour market in which there is a clear hierarchy of jobs. At the bottom are operative jobs, then come blue-collar, then managerial jobs. Managerial jobs pay the highest wages, and so are most desirable. Operative jobs, by contrast, are the least desirable. Blue-collar jobs pay the same wage as operative jobs, but are more desirable because they offer the prospect of promotion.

When workers enter the labour market they may seek to obtain either a blue-collar job or a managerial job. A crucial factor in determining the initial level at which workers enter will be their innate ability to perform managerial tasks. However the number of managerial jobs are constrained, so not all those seeking managerial posts will obtain one. The number of blue-collar jobs are also limited. Whilst queues exist for managerial jobs, we assume that the market for operative and blue-collar jobs clears, and the corresponding wage level accommodates this.

The ability of the worker is also an important factor in determining the subsequent career path of the individual. Those individuals with a high ability that were previously unable to find a managerial job will attempt to obtain one in the next period. Also crucial here will be the quality of the match between the worker and their firm, and the general match of the worker to the sector in which their firm is located. Those with bad matches will seek to move to jobs for which they are better suited.

In contrast to previous work in this area, we seek to examine what factors impede the progression of workers to jobs which suit their abilities and match qualities. In this context, we seek to incorporate sector specific shocks, which serve to knock individuals off of their chosen career paths and disrupt their plans. The structure of the model is set out in detail below.
3.1 Sectors and Firms

The economy which we consider consists of a single low-tech sector and two high-tech sectors. This is illustrated in Figure 1. The low-tech sector (denoted $B$) uses a simple production process and requires only unskilled operatives. Each firm in this sector employs only one worker. The two high-tech sectors (denoted 1 and 2) have a more sophisticated production process, and firms in this sector need to employ both a blue-collar worker and a manager.

![Diagram of sectoral structure](image)

**Figure 1:** The sectoral structure of the economy

The analysis that follows seeks to determine what factors determine the relative balance of employment between the low-skilled and the two high-skilled sectors. This will determine the career opportunities of the workers in the economy and condition their movement between jobs. We will then be able to examine how external shocks impact on this balance. However, before we can determine the relative demand for employment of the three sectors, we need to say something about the supply of workers.
3.2 The supply of workers

Operative and blue-collar jobs

We assume that the economy is populated by $N$ workers. As noted above, the tasks that need to be performed in the low-tech sector $B$ are simple, so we assume that all workers can perform these tasks, and all workers are equally productive in this sector, producing $b$ units of output per period. Further, this market clears, so a worker will always be able to find a job in this sector. In the high-tech sectors, workers differ in their productivity. The production processes here are more sophisticated, and workers differ both in their ability to perform the blue-collar tasks and in their ability to be managers in this sector.

The ability of a worker to perform blue-collar tasks in the high-tech sector depends on two factors. Firstly we assume that a worker’s particular aptitudes are more suited to one sector than another. The quality of the match between a worker and sector 1 is denoted $\sigma$, which can take only two values, $H$ (well-matched) and $L$ (poorly-matched). A worker who is a good match for sector 1 is always a bad match for sector 2 and vice versa. A worker’s sectoral match is fixed over time, and is decided by a random draw in the initial period. The probability that a worker is well matched to firms in sector 1 (and therefore poorly matched to firms in sector 2) is denoted by $\Pr(\sigma = H) = \tau$.

Secondly, we assume that the productivity of the worker in the high-tech sectors depends on the quality of the match between the worker and the firm, denoted $\phi$. Even within the same sector, firms differ in their ethos and organisation, and workers are better suited to work in some firms than others. The quality of this match can be either high $H$ or low $L$. $\phi$ is decided by another random draw, the outcome of which is independent of the sector match and made after a worker obtains a job. We denote the probability that a typical worker has a good firm match by $\Pr(\phi = H) = \pi$.

Let $\lambda_j(\sigma, \phi)$ indicate the productivity of a blue-collar worker in sector $j$. For tractability,
we make the additional assumption that:

$$\lambda_j \equiv \lambda_j(H, H) > \lambda_j(H, L) = \lambda_j(L, H) = \lambda_j(L, L) \equiv b, \quad j = 1, 2.$$ 

That is, for a blue-collar worker to be highly productive in the high-tech sector, then they must have both a good sector match and a good firm match. If they have a bad match, on either count, then the value of their output is identical to that which they would have produced if they had worked as an operative in the low-tech sector.\footnote{More general rankings of the relative productivities of the different combinations of match quality are possible. However for the exposition in this paper, the main feature is that those with high quality sector and firm matches are more productive than other blue-collar and operative workers.}

**Managerial jobs**

The high tech sector also offers the prospect of a managerial job. We assume that managerial skills are more generic than blue-collar skills and hence sectoral match quality is not important. The productivity of a worker as a manager in a high-tech sector is again assumed to depend on the quality of the firm match $\phi$, but we also assume that it depends on the worker’s innate ability $\alpha$, which can also be either high ($H$) with probability $p$, or low ($L$) with probability $1 - p$. The productivity of a manager in sector $j$ is denoted $\theta_j(\alpha, \phi)$. For tractability, we assume that

$$\theta_j \equiv \theta_j(H, H) > \theta_j(H, L) = \theta_j(L, H) = \theta_j(L, L) \equiv b, \quad j = 1, 2$$

Thus for a worker to be highly productive as a manager in the high-tech sector then he must have both high ability and be well matched with their firm.\footnote{Again, more general rankings of relative productivities are possible. For the model, the important feature is that those of high ability and with good firm matches are more productive than blue-collar workers in the high tech sector.}
3.3 Workers and the incentive to move firms

We assume that workers know their ability level \( \alpha \) but that their suitability for blue-collar work in a particular sector \( \sigma \) and quality of firm match \( \phi \) are not directly observable. Therefore workers do not know at the outset to which job they are best suited. The problem for firms is even more difficult, because we assume that firms also cannot directly observe a particular worker’s ability level.

Although the individual components affecting productivity are not directly revealed, at the end of each period the worker’s overall productivity is revealed to both the firm and the worker. Only those blue-collar workers with a good sectoral match and a good firm match will have a high productivity level. All other blue-collar workers will either be unsuited to their sector, or to their firm, or both. Similarly, only those managers who have high ability and who have a good firm match will have high productivity level.

If a worker is badly matched then, in the next period, they will re-enter the job market. This is either because they seek to find a job to which they are better suited, or because the firm in which they work fires them in the hope of finding a better replacement. The various possibilities for movement are illustrated in Figure 2.

We assume that the managerial labour market opens first. The managerial pool consists only of workers who have managerial ability \( \alpha = H \). This will include workers who quit their blue-collar and operative jobs and workers who quit or were fired from their previous managerial position because they were badly matched. Some managerial posts will be filled by internal promotions, while the remaining managerial posts will be filled externally.

After all managerial posts have been filled, the remaining workers without employment

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8 We show in Appendix B that, given the parameterisation of our model, high ability workers will always choose to enter the managerial market because their expected wage is higher than in a blue-collar or operative job. Conversely, low ability workers will never choose to enter the managerial market because their expected wage is lower than in a blue-collar or operative job.
enter the blue-collar/operative recruiting pool. The pool includes both those high ability workers who put themselves forward for a managerial position but were not hired, and those low ability workers who were not retained by their employers. Again, some fraction of these workers are hired as blue-collar workers. Finally, those remaining obtain operative jobs, with the wage paid to operatives accommodating this.

Thus, our model allows for a rich variety of worker movements between firms and jobs. Separations may be both voluntary and involuntary. Workers may be promoted within firms if they are well-matched with a firm and have managerial ability. Workers who separate may find new jobs at the same level or may be forced to move to a lower occupation.

In order to find out how many workers will be successfully matched to their preferred occupation and firm, and how many will move between occupations and firms, we need to calculate the distribution of firms in the economy according to their hiring and promoting

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9In Appendix C we show that workers in the blue-collar/operative recruiting pool cannot successfully signal either their ability or the sector in which they are well matched. We also show that firms cannot or do not wish to screen applicants.
decisions.

3.4 The sectoral composition of the economy

Low-tech sector

We assume that there are $M_B$ potential firms in the low-tech sector$^{10}$ Firms can each produce $b$ units of output and pay a wage $w_b$. Each firm is subject to an idiosyncratic shock to its non-labour costs $c_i$ each period, which are a draw from a uniform distribution with support in the interval $c_i = [0, \bar{c}_B]$. Any low-tech firm which can earn a positive expected profit in the next period will operate, while those making a loss will not operate (this will include the closure of some existing firms). The profits of a low-tech firm $i$ in sector $B$ are therefore$^{11}

$$
\pi_i = b - w_b - c_i \quad \text{i in sector} \ B. \quad (1)
$$

High-tech sector

There are $M_1$ and $M_2$ potential firms in high-tech sectors $j = 1, 2$ respectively. Each firm needs to hire one blue-collar worker and one manager to operate. We assume that managers are paid by a sharing contract that specifies that they receive a fraction $1 - \gamma$ of the firm’s profits, while the blue-collar worker receives a wage $w_b$. The firm’s output is the sum of the two workers’ productivities, $\lambda_i + \theta_i$. Again, we assume that a firm’s non-labour cost is determined each period by a random draw from a uniform distribution with support in the interval $c_i = [0, \bar{c}_j]$. All potential high-tech firms with a positive expected profit will operate. The profits of each active high-sector firm $i$ are given by

$$
\pi_i = \gamma(\lambda_i(\sigma, \phi) + \theta_i(\alpha, \phi) - w_b - c_i) \quad \text{i in sectors} \ 1, 2. \quad (2)
$$

$^{10}$The number of active firms in each sector is endogenously determined.

$^{11}$To simplify the notation we have dropped the time subscripts.
Actual and potential jobs

We impose a restriction on the model that the total number of workers in the economy is less than the number of potential jobs $N < M_B + 2(M_1 + M_2)$. This implies that, in each period, some firms will be inactive and that there will be full employment.

3.5 A taxonomy of firms

At the beginning of each period all firms learn about the size of their non-labour costs $c_i$. Given their expectations about the equilibrium blue-collar wage and managerial wage, they decide whether to be active or inactive during the next period.\(^\text{12}\) Note that a firm’s decision about whether or not to operate will depend on their current constellation of workers. For example, those firms which currently have a well-matched worker, and anticipate keeping them, will confront a different situation to firms that are not currently operational. Since a firm’s hiring decision will be determined by their previous employment configuration, it is useful to define a taxonomy of firms on this basis.

Low-tech firms

Firms in the low-tech sector may be divided into three types, shown in Table 2.

<table>
<thead>
<tr>
<th>Type</th>
<th>Worker-type</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B0$</td>
<td></td>
<td>Inactive</td>
</tr>
<tr>
<td>$B1$</td>
<td>$\alpha = H$</td>
<td>Worker leaves</td>
</tr>
<tr>
<td>$B2$</td>
<td>$\alpha = L$</td>
<td>Worker stays</td>
</tr>
</tbody>
</table>

Table 2: Firm types in the low-tech sector

The employment behaviour of low-tech sector firms is relatively simple because the productivity of workers in this sector is known with certainty. A low-tech firm $i$ will be active

\(^{12}\)Firms become inactive if next period’s expected profits are negative. This behaviour is consistent with long-term profit maximization because there are no fixed costs.
if, conditional on their draw of the non-labour cost, their profits (1) are positive. Let \( c^*_B \) denote the critical value of non-labour costs such that (1) is zero:

\[
    c^*_B = b - w_b
\]  

All firms that draw a cost less than \( c^*_B \) will operate. Those firms that are active will either employ a worker with high managerial ability or low managerial ability. Workers with high ability will quit the firm and enter the managerial recruitment pool. These firms will therefore have to hire a new worker. Workers with low ability will stay with the firm in the following period, provided that the firm remains active.

**High-tech firms**

The employment dynamics of high-tech firms are more complicated because they have a combination of well-suited and badly-suited blue-collar and managerial workers. Table 3 presents a complete description of all possibilities.

<table>
<thead>
<tr>
<th>Type</th>
<th>Blue-collar worker</th>
<th>Manager</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j0 )</td>
<td></td>
<td></td>
<td>Inactive</td>
</tr>
<tr>
<td>( j1 )</td>
<td>( \alpha = H, \phi = H, \sigma = H )</td>
<td>( \phi = H )</td>
<td>Worker will enter managerial market, manager will stay, firm hires new worker</td>
</tr>
<tr>
<td>( j2 )</td>
<td>( \alpha = L, \phi = H, \sigma = H )</td>
<td>( \phi = H )</td>
<td>Worker will stay, manager will stay</td>
</tr>
<tr>
<td>( j3 )</td>
<td>( \alpha = H, (\phi = L \text{ or } \sigma = L) )</td>
<td>( \phi = H )</td>
<td>Worker will enter managerial market, manager will stay, firm hires new worker</td>
</tr>
<tr>
<td>( j4 )</td>
<td>( \alpha = L, (\phi = L \text{ or } \sigma = L) )</td>
<td>( \phi = H )</td>
<td>Worker will be fired, manager will stay, firm hires new worker</td>
</tr>
<tr>
<td>( j5 )</td>
<td>( \alpha = H, \phi = H, \sigma = H )</td>
<td>( \phi = L )</td>
<td>Worker will be promoted, manager will leave, firm hires new worker</td>
</tr>
<tr>
<td>( j6 )</td>
<td>( \alpha = L, \phi = H, \sigma = H )</td>
<td>( \phi = L )</td>
<td>Worker will stay, manager will leave, firm hires new manager</td>
</tr>
<tr>
<td>( j7 )</td>
<td>( \alpha = H, (\phi = L \text{ or } \sigma = L) )</td>
<td>( \phi = L )</td>
<td>Worker will enter managerial market, manager will leave, firm hires new worker and manager</td>
</tr>
<tr>
<td>( j8 )</td>
<td>( \alpha = L, (\phi = L \text{ or } \sigma = L) )</td>
<td>( \phi = L )</td>
<td>Worker will be fired, manager will leave, firm hires new worker and manager</td>
</tr>
</tbody>
</table>

**Table 3:** Firm types in the high-tech sectors
Of those managers who are currently employed, only those who are well matched with their firm \( (\phi = H) \) will want to remain with the same employer (types \( j_1 \)–\( j_4 \)). Their firm will also want to retain them. If the manager is not well matched then they will re-enter the managerial hiring pool and try to obtain a better placement elsewhere (types \( j_5 \)–\( j_8 \)).

Of the blue-collar workers, those of a low ability but with a high-quality sector match and a high-quality firm match will be retained by their existing employer. In contrast, those of low ability that are badly matched to either their firm or sector are sacked by the firm (types \( j_4 \) and \( j_8 \)).

Some workers who are currently employed in blue-collar jobs are of high ability, and therefore potentially management material. Those who are well matched to their existing firm will be promoted if the firm has a vacant managerial position (type \( j_5 \)). Other high ability workers will quit and attempt to find a management job via the managerial hiring pool (firm types \( j_1 \), \( j_3 \) and \( j_7 \)).

Note that firms can change from one firm type to another as a result of worker movement and recruitment. For example, a firm may sack a badly matched individual and recruit someone who is better suited. Note also that all high tech firms would like to become firms of type \( j_2 \). This type of firm has both a perfectly matched blue-collar worker and a perfectly matched manager. It is therefore the most productive. Firms of this type would happily retain their existing workforce, and the workers would be happy to stay. However, some firms of type \( 2 \) will still draw an unfavourable non-labour cost which means that, despite their well-matched workforce, they are unprofitable. They will therefore close, and their workers must seek alternative employment. Their replacements will consist of those firms that have drawn a favourable non-labour cost, which will start operating, but may be of any of the potential firm types.

A high-tech firm \( i \) will be active if, conditional on its draw of the non-labour cost, its profits \([2]\) are positive. Again, let \( c^* \) denote the critical value of non-labour costs such
that (2) is zero. For firms in the low-tech sector this critical value is fixed, but in the high-tech sector the value will vary depending on what combination of workers the firm currently employs. Table 4 summarises these critical values.

<table>
<thead>
<tr>
<th>Firm type</th>
<th>Critical value of non-labour costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>j0</td>
<td>( c^*_j = E(\lambda) + E(\theta) - w_b )</td>
</tr>
<tr>
<td>j1</td>
<td>( c^*_j = E(\lambda) + \theta(H,H) - w_b )</td>
</tr>
<tr>
<td>j2</td>
<td>( c^*_j = \lambda(H,H) + \theta(H,H) - w_b )</td>
</tr>
<tr>
<td>j3</td>
<td>( c^*_j = E(\lambda) + \theta(H,H) - w_b )</td>
</tr>
<tr>
<td>j4</td>
<td>( c^*_j = E(\lambda) + \theta(H,H) - w_b )</td>
</tr>
<tr>
<td>j5</td>
<td>( c^*_j = E(\lambda) + \theta(H,H) - w_b )</td>
</tr>
<tr>
<td>j6</td>
<td>( c^*_j = \lambda(H,H) + E(\theta) - w_b )</td>
</tr>
<tr>
<td>j7</td>
<td>( c^*_j = E(\lambda) + E(\theta) - w_b )</td>
</tr>
<tr>
<td>j8</td>
<td>( c^*_j = E(\lambda) + E(\theta) - w_b )</td>
</tr>
</tbody>
</table>

Table 4: Critical values for firms in the high-tech sector

Note that the critical value of non-labour costs above which a firm will exit or not enter the market is highest for firms who have a well-matched worker and manager \( j_2 \). It is lowest for firms who are seeking both a new worker and a new manager \( j_0, j_7 \) and \( j_8 \).

3.6 Equilibrium

Now that we have described the firm recruitment strategies and worker movements, we are in a position to define an equilibrium. There are 21 different types of firm: three in sector \( B \) and 9 in each of sectors 1 and 2.

Definition 1 A long-run perfect foresight equilibrium consists of a 31-tuple

\((w_b, M_{B0} \ldots M_{B2}, M_{10} \ldots M_{18}, M_{20} \ldots M_{28}, c^*_{B}, c^*_1, \ldots c^*_4, c^*_1, \ldots c^*_4)\) such that:

(i) \( M_{kt} = M_{kt+1} = M_k \geq 0 \) for each \( k = B0 \ldots B2, 10 \ldots 18, 20 \ldots 28 \)

\(^{13}\)See Section A.3 for a derivation.
(ii) the labour market clears

\[ M_{B1} + M_{B2} + \sum_{j=1}^{2} 2 \left( \frac{M_{j1} + M_{j3} + M_{j4} + M_{j5} + M_{j2} + M_{j6} + M_{j7} + M_{j8}}{M_{j2} + M_{j6} + M_{j7} + M_{j8}} \right) = N \]  

Condition (i) means that, in equilibrium, the number of firms of each type is constant and is non-negative. This implies that the number of each job type and the wage will also be constant from one period to the next. Out of equilibrium, the number of each firm type will change. Condition (ii) means that, in equilibrium, the labour market clears. The number of workers employed in the low-tech and the two high-tech sectors must be equal to the number of workers offering themselves for employment, \( N \).

### 3.7 Solving for equilibrium

In steady-state equilibrium, the number of each type of low-tech and high-tech firms operating in the economy will be constant. To find the equilibrium of the model we therefore adopt the following strategy. First, we define the dynamics of each firm type, relating the number of a particular firm type in period \( t + 1 \) to the number of firms belonging to each type in period \( t \). Secondly, we solve the system for the situation when the numbers of each firm type are constant between periods. The details of this solution are given in Appendix A.

Although a full algebraic solution to the model is not possible, the model can be solved numerically if values are chosen for the exogenous parameters. Although our model is clearly stylised, we have calibrated our model so that the predicted job reallocation rates roughly match rates reported by authors such as Haltiwanger, Scarpetta & Schweiger (2008). Job turnover rates are about 15% in the high-tech sectors and 39% in the low-tech sector. Our baseline specification is given in Table A.1.

Note that, although in equilibrium the relative number of each firm type stays constant,
firms are still entering and exiting the market, and are changing from one type to another. This is because firms draw a random non-labour cost each period. This accords well with the literature on job creation and job destruction in which firms in the same narrowly-defined sector are simultaneously creating and destroying jobs (Davis et al. 1996). Because job creation and destruction occurs even in equilibrium, there is also worker movement in equilibrium as individuals are forced to find new employment. Workers also move voluntarily in an attempt to find the firms with whom they are best suited.

4 A comparative static analysis of the model

We are now in a position to consider the comparative static properties of the model. In what follows we focus in particular on two types of shock:

1. Changes in the productivity of different types of worker
2. Changes in the non-labour costs of the different sectors.

We focus on these effects because they correspond to two notions that have received much attention in the theoretical and empirical literature: “skill-biased technological change” and “sector-biased technological change”. We model the former by changes in the parameters $\theta$, $\lambda$ and $b$. We model the latter by looking at changes in the relative costs of production in each high-tech sector.

We focus on five sets of derived variables which have clear empirical counterparts:

1. Job creation and destruction rates. In our model job destruction occurs when firms leave the market. High-tech firms which exit destroy two jobs, whilst low-tech firms which exit destroy only one. Other worker-firm separations are not job destruction because the jobs continue to exist. In equilibrium, job creation equals job destruction.
The job turnover rate is twice the job destruction or job creation rate. All are expressed as a proportion of total jobs in that sector.

2. *Worker turnover rates.* Worker turnover occurs either when jobs are destroyed/created, or when workers and firms separate. Worker turnover is therefore always at least as big as job turnover. It includes both voluntary and involuntary movements between firms, but does not include promotion, which is a within-firm movement.

3. *Promotion rates.* Promotions are worker movements from blue-collar to managerial positions within the firm, and only occur in firms which separate from their current manager. Only well-matched, high-ability blue-collar workers can be promoted. The promotion rate is expressed as a proportion of the number of high ability workers moving.

4. *Relative wages.* Relative wages are expressed as the ratio of the managerial wage (which is a share of firm profits) and the blue-collar wage $w_b$. There is, in fact, a range of managerial wages in equilibrium, since these depend on the profitability of the firm. To simplify the discussion we discuss the average managerial wage and, in order to assess within-group wage inequality, we also present results for the variance of the managerial wage.

5. *Mismatch.* Our model allows us to consider two different types of mismatch. First, we consider what fraction of high ability workers are not employed in managerial positions. This will simply be a function of the size of the high-tech sector, since all firms in this sector employ a manager. Second, we consider what fraction of the remaining workers are not employed in the sector to which their skills are best-suited.

Table 5 summarizes our comparative statics results. Before we analyze these in detail we make three general observations. First, comparing across rows we see that job turnover and promotion rates are positively correlated. This prediction is consisted with the preliminary
evidence presented in Figure 3, which shows a positive relationship between promotion and job turnover rates in UK firms.

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_1$</th>
<th>$\theta_1$</th>
<th>$b$</th>
<th>$\bar{c}_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job creation/destruction rate in sector $B$</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>Job creation/destruction rate in sector 1</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Job creation/destruction rate in sector 2</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Worker turnover rate in sector $B$</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>Worker turnover rate in sector 1</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Worker turnover rate in sector 2</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Promotion rate sector 1</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Promotion rate sector 2</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Blue-collar/operative wage</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Relative wage sector 1</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>Relative wage sector 2</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>Variance of managerial wage sector 1</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>Variance of managerial wage sector 2</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>Mismatch of high ability workers</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Mismatch of workers to sector 1</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Mismatch of workers to sector 2</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>−</td>
</tr>
</tbody>
</table>

Table 5: Comparative statics: selected parameters

Second, in general our model predicts a positive correlation between the average sectoral wages and their corresponding variance. This prediction is also supported by the data. In Figure 4 we plot the mean hourly wage against the standard deviation of the hourly wage and find that they are strongly positively correlated both for manufacturing ($\rho = 0.55$) and services ($\rho = 0.71$).

Third, we observe that when we increase the range of the idiosyncratic shocks (a rise in $\bar{c}_1$) both the job turnover rate and the variance of wages in sector 1 increase. This prediction of our model is in agreement with the empirical finding of Kambourov & Manovskii (2008a), who suggest that an increase in the variability of productivity shocks to occupations has resulted in higher job turnover rates and higher inequality in the United States.
Figure 3: Correlation of promotion and worker-turnover rates within industries 1992–2004. 
Source: authors’ calculations from the BHPS. The turnover rate is calculated as the proportion of workers who are not working for the same employer in consecutive years. The promotion rate is calculated as the proportion of workers who change from a non-managerial to a managerial position in consecutive years.

Figure 4: Correlation of mean and variance of real wages, sectors 1 and 2. Source: authors’ calculations from the BHPS.
4.1 Changes in the productivity of well-matched managers ($\theta_1$)

An increase in the relative productivity of their managers ($\theta_1$) will increase the expected profitability of high-tech firms in sector 1. This will mean that such a firm can have a higher level of non-labour costs than before and still decide to operate. As a consequence, the number of viable high-tech firms in sector 1 will increase and they will employ more managers and more blue-collar workers. Hence the demand constraint on managerial and blue-collar jobs in the economy will be relaxed. Such a change will be particularly beneficial to high ability workers, since the number finding themselves stuck in blue-collar (or operative) positions will decline. This increase in demand will, however, have a knock on effect for firms in other sectors, which must now pay their blue-collar/operative workers higher wages.

The increase in $\theta_1$ has important consequences for equilibrium job flows in the economy. The equilibrium rate of job destruction in sector 1 will decrease, as the proportion of firms that are unprofitable, consequent on the drawing of their non-labour cost, will decline. Workers in this sector will therefore benefit in terms of greater job security. However the rate of job turnover in the other high-tech sector and the low-tech sector will increase. This is because firms in this sector must now pay their blue-collar/operative workers a higher wage, without any compensating rise in productivity, and so are more vulnerable to fluctuations in non-labour costs.

The pattern of change for worker turnover rates is qualitatively similar to that for job turnover rates. However, the worker turnover rate in the sector 1 falls faster than the job turnover rate, and therefore the rate of “churning” (the gap between the worker turnover and job turnover) declines. This is because the number of well matched firms increases. This is illustrated in Figure 5. It is interesting to note that the rate of promotion also falls as the number of well matched firms increases. The opposite is true in sector 2.

In our model, the low-tech sector serves to soak up excess labour. Since the demand for
Figure 5: The impact of changes in $\theta_1$ on job and worker turnover by sector

Figure 6: The impact of changes in $\theta_1$ on wages by sector
Mismatch of high ability workers
Mismatch of workers to sector 1
Mismatch of workers to sector 2
Mismatch of high ability workers

Figure 7: The impact of changes in $\theta_1$ on mismatch

managers and blue-collar workers has increased, the number of workers requiring operative jobs has fallen. The wages paid to the remaining operatives and hence blue-collar workers will increase as a consequence. However the wages of blue-collar workers do not increase as much as the wages of managers in sector 1, who receive a proportion of the increased firm profits. Thus, technological change which makes managers more productive increases their wages wages relative to blue collar workers. This is illustrated in Figure 6. Note that the relative wages of managers declines in sector 2. Firms in this sector have become relatively less profitable and their managers suffer both absolute and relative wage declines as a consequence.

A direct consequence of the reduction in the number of firms in the operative sector is a reduction in the mismatch of high-ability workers (Figure 7). The increase in the productivity of managers in sector 1 increases the survival rate of firms in that sector and in particular of those firms with well matched workers. Given then that the destruction of good matches has declined, the level of blue-collar worker mismatch has also declined in that sector. In contrast, the change has a detrimental effect in the relative competitiveness of sector 2 with an associated increase in blue-collar mismatch.
4.2 Changes in the productivity of well-matched blue-collar workers ($\lambda_1$)

The consequences of an increase in the productivity of well-matched blue-collar workers are qualitatively similar to those of an increase in $\theta_1$. An increase in $\lambda_1$ increases the expected profitability of the high-tech firms in sector 1 and so increases the demand for both managers and blue-collar workers, with the consequent ramifications for the low-tech sector and sector 2. The intuition for the mismatch results is similar to that discussed above in relation to changes in the productivity of managers in sector 1.

An interesting point to note however is that an increase in the productivity of blue-collar workers sector 1 serves to increase wage inequality in sector 1. This is because the effect on the profitability of high-tech firms, and the positive impact that this has on managerial wages, outweighs the beneficial demand effects on blue-collar wages. Hence the relative wage of managers in sector 1 increases.

4.3 Changes in the productivity of operatives ($b$)

Increasing the productivity of operatives in the low-tech sector increases the relative size of that sector and reduces job destruction rates in low-tech firms. As before, the qualitative impact on worker turnover rates is the same as for job turnover rates, because the latter is a large part of the former. Figure 8 indicates that the rate of churning in the low-tech sector declines as worker turnover rates fall more rapidly than job turnover rates.

In the high-tech sector, the increase in $b$ raises the productivity of firms with badly matched blue-collar workers. Hence, more of this type of firm will survive. By contrast, high-tech firms with well matched blue-collar workers do not benefit in terms of productivity but must pay higher wages to their blue-collar workers. The proportion of such firms therefore declines. Hence the level of job turnover, churning and the rate of promotions will increase. The increase in $b$ will increase the blue-collar/operative wage and reduces the relative wage of managers.
Job and worker turnover rates

Sector 1

Productivity of operatives ($b$)

Sector 2

Worker turnover rate

Job turnover rate

Mismatch of high ability workers

Mismatch of workers to sectors 1 and 2

Figure 8: The impact of changes in $b$ on job and worker turnover by sector

Mismatch of high ability workers

Mismatch of workers to sectors 1 and 2

Figure 9: The impact of changes in $b$ on mismatch

29
Figure 9 shows that the increase in the size of the operative sector causes an increase in the mismatch of all types of workers. The increase in the mismatch of high-ability workers is due to the decline in managerial opportunities. As noted above, the increase in $b$ reduces the number of firms with good matches, leading to an increase in the mismatch of blue-collar workers. This change has a symmetric effect on the two high-tech sectors.

In summary, we might expect that increases in $b$ would have opposite comparative static effects to increase in $\lambda$, since rises in $b$ benefit the low-tech sector, while rises in $\lambda$ benefit the high-tech sector. For most comparisons this is indeed the case. However, note that increases in either parameter will increase the blue-collar/operative wage. This is because labour supply is fixed in our model and any increase in demand causes an increase in wages. Note that relative wages of managers fall in response to an increase in $b$ since there is now a larger proportion of firms with badly matched blue-collar workers and so the average productivity of high-tech firms has fallen.

### 4.4 Asymmetric change in non-labour costs ($\bar{c}_1$)

Finally, we consider an asymmetric shock which makes one of the high-tech sectors more profitable than the other. This corresponds to a sector-biased technological change. We model this by considering a change in the distribution of non-labour costs, with sector 1 becoming relatively less profitable if $\bar{c}_1$ rises. The effects of an increase in the upper interval of non-labour cost is to reduce the expected profitability and hence the size of the sector concerned. This will lessen the demand for managers and blue-collar workers in that sector.

Sector 1 is now smaller and has higher job creation and destruction rates than sector 2 because it has a smaller number of well-matched firms. The first panel of Figure 10 shows that as $\bar{c}_1$ increases both job and worker turnover increase, but the latter increases faster and so sector-biased technological change causes an increase in churning in the sector.
which experiences increases in costs. However, because there is now a higher number of 
badly-matched firms (including those of type 5) the promotion rate will actually increase.

Figure 11 shows how wages respond to an increase in \( \bar{c}_1 \). As sector 1 shrinks, releasing 
workers of both high and low ability, the wages of blue-collar/operatives will be driven 
down. The wages of managers in sector 1 will however fall more than the wages of blue-
collar workers, so relative wages will also decline. Note by contrast that relative wages in 
sector 2 increase in line with sector profits.

![Figure 10: The impact of changes in \( \bar{c}_1 \) on job and worker turnover by sector](image1)

![Figure 11: The impact of changes in \( \bar{c}_1 \) on wages by sector](image2)
Mismatch of high ability workers
Mismatch of workers to sector 1
Mismatch of workers to sector 2

Non-labor cost in sector 1 ($c_1$)

Mismatch of workers in sector 1
Mismatch of workers in sector 2
Mismatch of high ability workers

Figure 12: The impact of changes in $c_1$ on mismatch

The deterioration in the average productivity of firms in sector 1 boosts the size of the operative sector, which in turn increases the mismatch of high-ability workers, shown in Figure 12. The disproportionately higher destruction of jobs by well-matched firms has a positive impact on the level of mismatch in sector 1. In contrast, the relative improvement in the competitiveness of sector 2 has reduced its level of mismatch.

5 Conclusions

There is a large and growing literature devoted to the development of theoretical models that attempt to explain the complex interaction of job reallocation patterns with the patterns of worker turnover. In advanced economies firms and workers are subject to a variety of shocks (technological, structural, cyclical and idiosyncratic) and new jobs are continuously created and destroyed. Workers make frequent job to job movements and often change their sector of employment. These moves are often voluntary but can also be involuntary, especially when workers are forced to accept lower paying jobs during their lifetime workers also move up and down the career job ladder. These steps can take place within a firm or as workers move to new firms and even new sectors.
To our knowledge there has been no attempt to integrate the reallocation and the job career literatures and, as we argue in this paper, there are good reasons to do so. Intuition suggests that workers are more likely to be promoted when other firms are also more likely to offer them jobs. If this is the case then we would expect to observe a positive correlation between job creation and promotion rates and this is exactly what the data suggests. We have developed and analyzed a dynamic, stochastic, multi-sectoral equilibrium model that allows for worker turnover, job turnover and career mobility.

We have then examined the impact of a rich variety of potential economic shocks: from sectoral to economy-wide ones and from skill-biased technological shocks to others that affect all workers uniformly. Our model makes two novel predictions that seem to be supported by the data. Firstly, there is a positive correlation between job turnover rates and promotion rates, Secondly, there is a positive correlation across sectors between mean real incomes and their corresponding variance. In addition, our model suggests that a sectoral shock that adversely affects the profitability of that sector will increase both its job turnover rate and its income inequality, a prediction that is consistent with recent empirical findings; see Kambourov & Manovskii (2008a). A challenging opportunity for future research would be to embed our model within a search-theoretic framework, e.g. Kiyotaki & Lagos (2007), so that unemployment related issues can also be addressed.

References


Appendix A  Finding the equilibrium

A.1 Firm Dynamics

The equilibrium of the model is characterised by a steady state in the number of each type of low-tech and high-tech firms operating in the economy. To find the equilibrium of the model we proceed in a number of stages. First we define the dynamics of each firm type, relating the number of a particular firm type in period $t + 1$ to the number of the particular firm type in period $t$. Secondly, we solve the system for the situation when the numbers of each firm type are constant between periods.

Low-tech firms

Beginning with the low-tech sector, the equations which define the dynamics of the different types of low-tech firms are as follows:

$$M_{B1t+1} = \frac{c^*_B}{c_B} \times \{\Pr(\alpha = H \mid B)\} \times M_B \quad (A.1)$$

$$M_{B2t+1} = \frac{c^*_B}{c_B} \times \{\Pr(\alpha = L \mid B)\} \times M_B \quad (A.2)$$

and

$$M_{B0t+1} = M_B - M_{B1t+1} - M_{B2t+1} \quad (A.3)$$

where $\Pr(\alpha = H \mid B)$ denotes the probability that a worker has high ability, given that they are searching for a job in the blue-collar market. We can define the complementary probability, $\Pr(\alpha = L \mid B)$, in a similar way.\(^{14}\)

Equation (A.1) states that the number of active firms in the low-tech sector employing a high ability blue-collar worker at time $t + 1$ is equal to the fraction of firms in the low-tech

\(^{14}\)In fact, these probabilities are time-dependent but since we are only interested in steady-state solutions we have dropped the time subscript for notational simplicity.
sector which at the beginning of the period have drawn relatively low values for non-labour costs and have hired a high ability worker from the blue-collar market. Equation (A.2) is the corresponding expression for those active firms that hired a low ability worker. The number of inactive firms in the low-tech sector (A.3) is obtained by subtracting the number of active firms from the mass of firms in that sector.

High-tech firms

In the high-tech sector there are eight different firm types. Denoting the productivity of a well-matched blue-collar worker as \( \lambda^H_j = \lambda_j(H, H) \) and the productivity of a badly-matched worker as \( \lambda^L_j = \lambda_j(H, L) = \lambda_j(L, H) = \lambda_j(L, L) \), we have:

\[
M_{j1t+1} = \frac{c^*_{j1}}{c_j} \cdot \{ \Pr(\alpha = H \cap \lambda_j = \lambda^H_j) \mid B \} \cdot (M_{j1t} + M_{j3t} + M_{j4t} + M_{j5t}) \\
+ \frac{c^*_{j4}}{c_j} \cdot \pi \cdot \{ \Pr(\alpha = H \cap \lambda_j = \lambda^H_j) \mid B \} \cdot (M_{j7t} + M_{j8t} + M_{j0t}) \tag{A.4}
\]

\[
M_{j2t+1} = \frac{c^*_{j1}}{c_j} \cdot \{ \Pr(\alpha = L \cap \lambda_j = \lambda^L_j) \mid B \} \cdot (M_{j1t} + M_{j3t} + M_{j4t} + M_{j5t}) \\
+ \frac{c^*_{j2}}{c_j} \cdot M_{j2t} + \frac{c^*_{j3}}{c_j} \cdot \pi \cdot M_{j6t} \\
+ \frac{c^*_{j4}}{c_j} \cdot \pi \cdot \{ \Pr(\alpha = L \cap \lambda_j = \lambda^H_j) \mid B \} \cdot (M_{j7t} + M_{j8t} + M_{j0t}) \tag{A.5}
\]

\[
M_{j3t+1} = \frac{c^*_{j1}}{c_j} \cdot \{ \Pr(\alpha = H \cap \lambda_j = \lambda^L_j) \mid B \} \cdot (M_{j1t} + M_{j3t} + M_{j4t} + M_{j5t}) \\
+ \frac{c^*_{j3}}{c_j} \cdot \pi \cdot \{ \Pr(\alpha = H \cap \lambda_j = \lambda^L_j) \mid B \} \cdot (M_{j7t} + M_{j8t} + M_{j0t}) \tag{A.6}
\]
Equation (A.4) shows that firms of type $j_1$ (that employ a well-matched manager and a well-matched worker who has high ability) may come from a number of different sources: Firms that keep their high ability manager but replace their blue-collar worker ($j_1, j_3, j_4, j_5$) may, if they remain active, employ a blue-collar worker with the necessary characteristics. This is captured by the first term in (A.4); Firms that hire both a manager and a blue-
collar worker \((j0, j7, j8)\) may also end up as type \(j1\) firms.\(^{15}\) The probability of this event is equal to the probability that the manager has a high-quality firm match, \(\pi\), multiplied by the probability that a hired blue-collar worker has high ability, a good firm match and a good sector match.\(^{16}\)

The formula for firms of type \(j2\) \(^{(A.5)}\) is similar for that of type \(j1\): Firms that keep their high ability manager but replace their blue-collar worker \((j1, j3, j4, j5)\) may, if they remain active, become a \(j2\) firm if they employ a low ability blue-collar worker. This is captured by the first term in \((A.5)\); Firms that hire both a manager and a blue-collar worker \((j0, j7, j8)\) may also end up as type \(j2\) firms; Existing \(j2\) firms that draw a favourable non-labour cost remain as \(j2\) firms, reflected in the second element of \((A.5)\); Finally, firms of type \(j6\) already possess a blue-collar worker of the necessary characteristics and so can become a \(j2\) firm if they recruit a high ability, well-matched manager.

The interpretation of \((A.6)\) and \((A.7)\), which correspond to the dynamics of \(j3\) and \(j4\) firms, may be interpreted in a similar way.

The dynamics of types \(j5\)-\(j8\) are relatively simple, and are shown by equations \((A.8)\) to \((A.11)\). These types of firm have a manager with a low-quality firm match, who was hired in the last recruitment round. Hence these firms must derive from firms of type \(j7\), \(j8\) and \(j0\), and cannot derive from firms of type \(j1 - j4\).\(^{17}\)

It is also only firms of type \(j7\), \(j8\) and \(j0\) that can become firms of type \(j7\) or \(j8\) since, assuming that they are active, only these types of firm replace both of their workers. Other firm types cannot end up with such an unfortunate configuration of badly matched workers.

Finally, the number of inactive firms in sector \(j\) \(^{(A.12)}\) can be found by subtracting the number of active firms from the total number of potential firms in that sector.

\(^{15}\) Notice that this second group includes firms that were inactive last period.

\(^{16}\) Remember that in our model the managerial market clears first.

\(^{17}\) Note that firms of type \(j5\) cannot remain of this type, as they will promote their blue-collar worker and, as a result, will have a high ability, well matched manager.
A.2 The probability of hiring different types of worker

The equations describing the dynamics of the different types of firm contain 6 conditional probabilities relating to the probability of hiring particular worker types, which we shall now derive.

Let \( h_t \) denote the number of high ability workers who do not have managerial jobs in period \( t \), and who are therefore looking for a job in the blue-collar market. Then:

\[
h_t = pN - \sum_{j=1}^{2} \sum_{i=1}^{8} M_{jit} \tag{A.13}
\]

where \( p \) is the fraction of high ability workers in the population. Equation (A.13) states that the number of high ability workers who do not have managerial jobs can be found by subtracting the number of active firms in the two high-tech sectors (each one employs one high ability worker) from the total number of high ability workers.

Now let \( s_{1t} \) denote the number of workers who are well-suited to working in sector 1, but who do not have a job in sector 1, and who are therefore looking for a job in the blue-collar market:

\[
s_{1t} = \tau N - \tau \sum_{j=1}^{2} \sum_{i=1}^{8} M_{jit} - (1 - \tau)M_{15t} - M_{12t} - M_{16t} \tag{A.14}
\]

The total number of workers who are well-suited to working in sector 1 is \( \tau N \), and we subtract from this the number of these workers who are not looking for a job in the blue-collar market. This includes a fraction \( \tau \) of all workers in managerial jobs, and a further \((1-\tau)\) managers in sector 5, who must also be well-matched because they were promoted by their firm. It also includes the blue-collar workers in firm types 2 and 6.

In a similar way, the number of workers who are well-suited to working in sector 2, but who are looking for a job in the blue-collar market is:

\[
s_{2t} = (1 - \tau)N - (1 - \tau) \sum_{j=1}^{2} \sum_{i=1}^{8} M_{jit} - \tau M_{25t} - M_{22t} - M_{26t} \tag{A.15}
\]
This means the total number of workers who are looking for a job in the blue-collar market is \( \beta_t = s_1t + s_2t \):

\[
\beta_t = N - \sum_{j=1}^{2} \sum_{i=1}^{8} M_{j1t} - M_{12t} - (1 - \tau)M_{15t} - M_{16t} - M_{22t} - \tau M_{25t} - M_{26t} \quad (A.16)
\]

Using the above expressions we can proceed with the derivations of the 6 conditional probabilities:

\[
\Pr(\alpha = H \mid B) = \frac{h_t}{\beta_t} \quad (A.17)
\]

\[
\Pr(\alpha = L \mid B) = 1 - \frac{h_t}{\beta_t} \quad (A.18)
\]

\[
\Pr(\alpha = H \cap \lambda_j = \lambda_j^H) \mid B = \pi \frac{h_t s_{jt}}{(\beta_t)^2} \quad (A.19)
\]

\[
\Pr(\alpha = L \cap \lambda_j = \lambda_j^H) \mid B = \frac{h_t (\beta_t - h_t) s_{jt}}{(\beta_t)^2} \quad (A.20)
\]

\[
\Pr(\alpha = H \cap \lambda_j = \lambda_j^L) \mid B = \frac{h_t (\beta_t - \pi s_{jt})}{(\beta_t)^2} \quad (A.21)
\]

\[
\Pr(\alpha = L \cap \lambda_j = \lambda_j^L) \mid B = \frac{(\beta_t - h_t)(\beta_t - \pi s_{jt})}{(\beta_t)^2} \quad (A.22)
\]

### A.3 The expected profits of firms

In determining whether a particular firm will operate in the next period, the firm’s drawing of non-labour cost is crucial, as this, in combination with the firm’s labour costs and productivity will determine whether or not the firm is profitable. In order to solve the model for the equilibrium number of firm types, we need to express the profitability of each firm type in terms of these quantities.

Firms of type \( j1, j3, j4 \) and \( j5 \) only hire a blue-collar worker and the expected profits of such a firm \( i \) from \( (2) \) is given by:

\[
\pi_i = \gamma (E(\lambda_j(\sigma, \phi)) + \theta_j(H, H) - w_b - c_i) \quad (A.23)
\]
Let $c^*_j$ denote the critical value of non-labour costs such that this expression vanishes:

$$c^*_j = E(\lambda_j(\sigma, \phi)) + \theta_j(H, H) - w_b$$  \hfill (A.24)

Firms of type $j2$ keep both their blue-collar worker and their manager, so their profits are given by

$$\pi_i = \gamma(\lambda_j(H, H) + \theta_j(H, H) - w_b - c_i)$$  \hfill (A.25)

Let $c^*_{j2}$ denote the corresponding critical value:

$$c^*_{j2} = \lambda_j(H, H) + \theta_j(H, H) - w_b$$  \hfill (A.26)

Firms of type $j6$ fire their manager but retain their blue-collar worker and so their expected profits are given by

$$\pi_i = \gamma(\lambda_j(H, H) + E(\theta_j(H, \phi)) - w_b - c_i)$$  \hfill (A.27)

with a critical value denoted by $c^*_{j3}$:

$$c^*_{j3} = \lambda_j(H, H) + E(\theta_j(H, \phi)) - w_b$$  \hfill (A.28)

Finally, firms of type $j7$ and $j8$ need to hire both blue- and white-collar workers and thus the expected profits of a typical firm $i$ is given by:

$$\pi_i = \gamma(E(\lambda_j(\sigma, \phi)) + E(\theta_j(H, \phi)) - w_b - c_i)$$  \hfill (A.29)

with a critical value denoted by $c^*_{j4}$:

$$c^*_{j4} = E(\lambda_j(\sigma, \phi)) + E(\theta_j(H, \phi)) - w_b$$  \hfill (A.30)
A.4 The expected productivity of new hires

The expressions \[A.23, A.27, A.29\] include expressions for the expected productivity of the new hires. For blue-collar workers we have:

\[
E(\lambda_j(\sigma, \phi)) = \{\Pr(\lambda_j = \lambda_j^H \mid B) \cdot \lambda_j^H + (1 - \{\Pr(\lambda_j = \lambda_j^H \mid B)\} \cdot b
\]

\[
= \pi \frac{s_{jt}}{\beta_t} \times \lambda_j + \left(1 - \pi \frac{s_{jt}}{\beta_t}\right) \times b \quad (A.31)
\]

while for managers we have:

\[
E(\theta_j(H, \phi)) = \pi \cdot \theta_j(H, H) + (1 - \pi) \cdot b \quad (A.32)
\]

Appendix B Separation of markets

In order to simplify the analysis, we require that only high ability workers enter the managerial pool. This is ensured if (1) the expected wage of high ability workers is greater if they enter the managerial pool, and (2) if the expected wage of low ability workers is greater if they do not enter the managerial pool.

Condition 1

Expected profits in sector \(j = 1, 2\) are given by:

\[
E(\Pi_j) = \frac{c_{j1}}{2} \frac{M_{j1}}{M_j - M_{j0}} + \frac{c_{j2}}{2} \frac{M_{j2}}{M_j - M_{j0}} + \frac{c_{j3}}{2} \frac{M_{j6}}{M_j - M_{j0}} + \frac{c_{j4}}{2} \frac{M_{j7} + M_{j8}}{M_j - M_{j0}}
\]

The marginal firm in each sector earns zero profit. The most profitable firm earns profits equal to \(c^*\) (see Table 4). Because costs have a uniform distribution, the average firm
earns profits of $c^*/2$. The expected profits in the two high-tech sectors taken together, $E(\Pi)$, are given by:

$$E(\Pi) = \frac{E(\Pi_1)(M_1 - M_{10}) + E(\Pi_2)(M_2 - M_{20})}{M_1 - M_{10} + M_2 - M_{20}}$$

Since managers are compensated with a share of profits $\gamma$, we require that:

$$\gamma E(\Pi) > w_b$$

**Condition 2**

We require that:

$$w_b > \gamma (2b - w_b - E(c))$$

where, $E(c)$ is the average level of non-labour cost in the two high-tech sectors (where uniformity implies $E(c) = E(\Pi)$), and so the right hand side is the expected wage of a low ability worker as a manager.

In our calibration we choose parameter values to ensure that both of these conditions are satisfied.

**Appendix C  Pooling of workers**

For simplicity, we require that firms do not have an incentive to hire high ability workers for blue collar jobs. We therefore require that the expected profits from hiring a high ability worker are lower than expected profits from hiring a randomly drawn worker.

When firms in sector $j$ hire a high ability worker for a blue-collar job there are three possible outcomes: Firstly, the manager has a good firm match with probability $\pi$ and so the blue-collar worker must leave the firm to try and get a managerial position at another
firm. This yields expected profits:
\[ \pi \cdot \frac{c_{j1}}{2} \]

Secondly, the manager leaves with probability \( 1 - \pi \) and the worker is promoted. Firms will only promote workers if their output as blue-collar workers is high, which occurs if \( \sigma = H \) and \( \phi = H \) with probability \( \tau \pi \). Expected profits in this case are:
\[ (1 - \pi) \cdot \tau \pi \cdot \frac{c_{j1}}{2} \]

Thirdly, the manager leaves and the worker is not promoted, in which case they leave in order to try and get a managerial position elsewhere. Expected profits in this case are:
\[ (1 - \pi) \cdot (1 - \tau \pi) \cdot \frac{c_{j4}}{2} \]

Hence, the expected profits from hiring a high ability worker for a blue-collar job are given by:
\[ E(\Pi_h) = \pi \cdot \frac{c_{j1}}{2} + (1 - \pi) \cdot \tau \pi \cdot \frac{c_{j1}}{2} + (1 - \pi)(1 - \tau \pi) \cdot \frac{c_{j4}}{2} \]

Now consider the case where the firm does not attempt to screen high ability workers for the blue-collar job. In this case there are five possible outcomes: Firstly, the manager stays with probability \( \pi \) and the worker also stays because they are low ability and well matched to the firm and the sector with probability \( (1 - p) \cdot \tau \pi \). This yields expected profits:
\[ \pi \cdot (1 - p) \cdot \tau \pi \cdot \frac{c_{j2}}{2} \]

Secondly, the manager stays and the worker leaves, with expected profits:
\[ \pi \cdot (1 - (1 - p) \cdot \tau \pi) \cdot \frac{c_{j1}}{2} \]
Thirdly, the manager leaves with probability $1 - \pi$ and the worker is promoted. In this case, workers are only promoted if they are high ability and well matched to the sector and the firm, with overall probability $p\tau\pi$, in which case expected profits are:

$\left(1 - \pi\right) \cdot p\tau\pi \cdot \frac{C_{j1}}{2}$

Fourthly, the manager leaves and the blue-collar worker remains in that position, implying profits of:

$\left(1 - \pi\right) \cdot (1 - p)\tau\pi \cdot \frac{C_{j3}}{2}$

Finally, both the manager and the blue-collar worker leave yielding profits of:

$\left(1 - \pi\right) \cdot (1 - \tau\pi) \cdot \frac{C_{j4}}{2}$

Hence the expected profits hiring a randomly drawn worker for a blue-collar job are:

$$E(\Pi_l) = \pi(1 - p)\tau\pi \frac{C_{j2}}{2} + \pi(1 - (1 - p)\tau\pi) \frac{C_{j1}}{2} + (1 - \pi)p\tau\pi \frac{C_{j4}}{2} + (1 - \pi)(1 - p)\tau\pi \frac{C_{j3}}{2} + (1 - \pi)(1 - \tau\pi) \frac{C_{j4}}{2}$$
We calibrate our model to ensure that $E(\Pi_h) < E(\Pi_l)$.

### Table A.1: Baseline parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_B$</td>
<td>Number of firms in low-tech sector $B^a$</td>
<td>100,000</td>
</tr>
<tr>
<td>$M_1$</td>
<td>Number of high-tech firms in sector 1$^a$</td>
<td>1,000</td>
</tr>
<tr>
<td>$M_2$</td>
<td>Number of high-tech firms in sector 2$^a$</td>
<td>1,000</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of workers</td>
<td>90,000</td>
</tr>
<tr>
<td>$b$</td>
<td>Productivity of badly-matched blue-collar worker and worker in low-tech sector</td>
<td>10</td>
</tr>
<tr>
<td>$\lambda_1(H,H)$</td>
<td>Productivity of well-matched blue-collar worker in sector 1</td>
<td>15</td>
</tr>
<tr>
<td>$\lambda_2(H,H)$</td>
<td>Productivity of well-matched blue-collar worker in sector 2</td>
<td>15</td>
</tr>
<tr>
<td>$\theta_1(H,H)$</td>
<td>Productivity of well-matched manager in sector 1</td>
<td>30</td>
</tr>
<tr>
<td>$\theta_2(H,H)$</td>
<td>Productivity of well-matched manager in sector 2</td>
<td>30</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Probability of a worker being well-matched to a firm in sector 1 (= probability of being badly matched to a firm in sector 2) $\Pr(\sigma = H)$</td>
<td>0.5</td>
</tr>
<tr>
<td>$p$</td>
<td>Proportion of workers with high managerial ability $\Pr(\alpha = H)$</td>
<td>0.30</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Probability of being well-matched to current firm $\Pr(\phi = H)$</td>
<td>0.4</td>
</tr>
<tr>
<td>$\bar{c}_B$</td>
<td>Upper support for distribution of costs in low-tech sector $B$</td>
<td>10</td>
</tr>
<tr>
<td>$\bar{c}_1$</td>
<td>Upper support for distribution of costs in high-tech sector 1</td>
<td>45</td>
</tr>
<tr>
<td>$\bar{c}_2$</td>
<td>Upper support for distribution of costs in high-tech sector 2</td>
<td>45</td>
</tr>
</tbody>
</table>

$^a$ Including inactive firms.
Appendix D  Mismatch of workers

Appendix A.2 provides us with expressions for the numbers of workers who are in jobs for which they are not well-matched. This includes high ability workers who are not in managerial positions, as well as workers who cannot find jobs in the sector to which they are well-matched.

The proportion of high ability workers who are mismatched because they are not in managerial positions is
\[ \frac{h_t}{pN}, \quad (D.1) \]
which follows from Equation (A.13).

The proportion of workers who are not working in sector 1, but who are well-matched to sector 1, is
\[ s_{1t} - \tau\left(\frac{M_{11t} + M_{13t} + M_{14t} + M_{15t} + M_{17t} + M_{18t}}{\tau((1-p)N + h_t)}\right), \quad (D.2) \]
which follows from Equation (A.14). Recall that \( s_{1t} \) is the number of workers who are well-suited to working in sector 1, but who are looking for a job. Of these, a fraction will find jobs in firms in sector 1. They cannot find jobs in sector 1 firms of types 2 and 6, because these firms have no job slots for blue-collar workers available. Thus, the number of workers who find a job in sector 1 is a fraction \( \tau \) of the remaining sector 1 firms, since they will be competing for these jobs with the \( 1 - \tau \) proportion of workers who are well-suited to jobs in sector 2. The denominator is the total number of workers who are suited to working in sector 1, including those who are low ability \( \tau((1-p)N) \) and those who are high ability but who cannot find a job as a manager \( \tau h_t \).

Similarly, the proportion of workers who are not working in sector 2, but who are well-matched to sector 2, is
\[ s_{1t} - (1 - \tau)\left(\frac{M_{21t} + M_{23t} + M_{24t} + M_{25t} + M_{27t} + M_{28t}}{(1 - \tau)((1-p)N + h_t)}\right), \quad (D.3) \]
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